SPARK-CHAMBER EXPERIMENTS: $\pi^- + p \rightarrow K^o + \Lambda^o$ AT 100 GEV

J. H. Smith
University of Illinois

July 10, 1968

In other study groups large spark-chamber magnets have been proposed.[1,2] These magnets are to be used as triggered "bubble chambers" with the magnetic-field region filled with detectors. Such a device is useful at very high energies, it is argued, because the usual reaction is of high multiplicity. This report singles out for analysis a particular reaction of low multiplicity $\pi^- + p \rightarrow K^o + \Lambda^o$ to investigate what types of magnets and what types of apparatus are necessary to measure the cross section in the forward peak as a function of angle at 100 GeV incident π^-. All aspects of the feasibility or interest in this experiment have not been investigated; it has been used merely as an example.

The configuration chosen for study is shown in Fig. 1. The pion beam is incident on a liquid hydrogen target 1 m long. The Λ^o decay is detected in a large spark-chamber system around the target, and the K^o decay into two charged π's is detected in a pair of spark chambers before and a pair of spark chambers after the magnet. The magnet allows a momentum measurement of the two decay pions. Their momenta plus a measurement of their opening angle allows the mass of the K^o to be reconstructed and its angle of production measured.

Rate and Beam Requirements

The π^- (p, Λ^o)K^o cross section has been measured at 8 GeV. It is 27 μbarns with about 22 μbarns in the forward peak. The shape of the forward cross section is exp (9.7 t) so that most of the peak lies at momentum transfers less than 4 pion masses. Assuming a Regge behavior $s^{2\alpha(0)-2}$ with $\alpha(0) = 1/2$ gives an expected cross section of 1.76 μbarns at 100 GeV. Only 2/3 of all Λ^o decay with charged products. Only 2/3 of the K^o's decay into charged pions and only 1/2 are short lived. Therefore $2/3 \times 2/3 \times 1/2 = 2/9$ of all decays are visible giving 0.39 μbarns as a visible cross section. If all of these could be seen, there would be $6 \times 10^{23} \times 0.07 \times 100 \times 0.39 \times 10^{-30} = 1.7 \times 10^{-6}$ events per incident π meson. A beam of 10^6 pions per pulse is adequate for the experiment, and it might be possible to pass such a beam directly through the chambers, but more likely they would have to be locally desensitized.

The beam presents no special problems.

Mass Resolution

If terms in the pion mass are neglected and angles assumed small then the mass of the K^o is given by

$$M^2 = p_1 p_2 \theta_{12}^2,$$

where p_1 and p_2 are the momenta of the decay pions, and θ_{12} is the opening angle between them. The error in the mass is

$$\frac{2\Delta m}{m} = \frac{\Delta p_1}{p_1} + \frac{\Delta p_2}{p_2} + \frac{2\Delta\theta_{12}}{\theta_{12}} .$$

Assuming that the two momentum error terms are equal, but uncorrelated, we get

$$\frac{\Delta m}{m} = \frac{\Delta p}{\sqrt{2}\, p} + \frac{\Delta\theta_{12}}{\theta_{12}} .$$

The error in the momentum can be made very small by making the magnet large, but there is little to be gained in making it much smaller than the angle-error term.

If $\Delta\phi$ is the characteristic accuracy to which the angle of a single track can be measured $\Delta\theta_{12} = \sqrt{2}\,\Delta\phi$ since the opening angle involves the independent measurement of the angle of two tracks.

The momentum is given by

$$p(\text{GeV}) = \frac{3}{100}\, B\,(\text{kG})\, r\,(\text{meters}),$$

and the radius of the track is determined by measuring its angle of bend α as it passes a distance L through the field. Thus,

$$p = \frac{3}{100}\,\frac{BL}{\alpha} .$$

Then,

$$\frac{\Delta p}{p} = -\frac{\Delta \alpha}{\alpha}.$$

The error in the angle of bend is the independent sum of the errors in the angles of a track before and after the magnet, or in magnitude

$$\frac{\Delta p}{p} = \frac{\sqrt{2}\,\Delta\phi}{\alpha}.$$

Finally,

$$\frac{\Delta m}{m} = \left(\frac{1}{\alpha} + \frac{\sqrt{2}}{\theta_{12}}\right)\Delta\phi.$$

To make the two errors comparable it is necessary to make the angle of bend roughly equal to the opening angle divided by $\sqrt{2}$, or

$$p = \frac{3}{100}\,\frac{BL}{\alpha} = \frac{3\sqrt{2}}{100}\,\frac{BL}{\theta_{12}}.$$

A suitable magnet therefore has

$$BL = \frac{100}{3\sqrt{2}}\;p\theta_{12}.$$

Now $p\theta_{12}/2$ is the transverse momentum given to one of the pions in K^o decay or approximately 0.2 GeV. Therefore a suitable magnet has

$$BL = \frac{100}{3\sqrt{2}}\,0.2 = 11.5\ \text{kG}\cdot\text{m}.$$

Actually since the momentum measurement also involves one of the angles necessary for the opening angle, the momentum and angle measurements are correlated in such a way that this number is reduced somewhat to about 7 kG·m.

This number is quite small, and it is worthwhile pointing out that since transverse momentum in any particle decays are of the order of 0.2 GeV that the result is general. Magnets of this size will be useful in the laboratory. Perhaps a 20 kG·m magnet would be a good, modest choice.

If a mass peak 20 MeV wide is sufficient to identify K^o's unambiguously,

$$\frac{\Delta m}{m} = \frac{10}{500} = \sqrt{2} \quad \frac{\sqrt{2}\,\Delta\phi}{\theta_{12}} = \frac{2\Delta\phi}{\theta_{12}}.$$

Then,

$$\Delta\phi = \frac{1}{100}\,\theta_{12}.$$

At 100 GeV θ_{12} = 8 mrads so that the necessary angular resolution is

$$\Delta\phi = 0.08 \text{ mrads.}$$

Transverse Momentum Resolution

The transverse momentum of the K^o is given by

$$p_1\theta_1 + p_2\theta_2 = p_\perp ,$$

$$\Delta p_{\perp} = p_1\Delta\theta_1 + \theta_1\Delta p_1 + p_2\Delta\theta_2 + \Delta p_2\theta_2$$

$$= \sqrt{2}\, p\Delta\theta + \sqrt{2}\, \theta\Delta p .$$

As in the section on mass resolution

$$\Delta p_{\perp} = \sqrt{2}\, p\Delta\phi + \frac{\sqrt{2}\, \theta p \sqrt{2}\, \Delta\phi}{\alpha} .$$

The two terms are comparable when $\alpha = \sqrt{2}\, \theta$ since θ for one particle is, on the average, $\theta_{12}/2$ in the forward direction the two terms are equal when $\alpha = \theta_{12}/\sqrt{2}$, i.e. exactly the same criterion as for the mass resolution.

If the two sources of error are made equal, they add independently and

$$\Delta p_{\perp} = \sqrt{2} \times \text{(first term)} = 2p\Delta\phi .$$

The experiment would have sufficient accuracy for most purposes if the forward peak out to 4 pion masses were to be broken up into 20 angular intervals. Thus

$$\Delta p_{\perp} \sim \frac{0.14 \text{ GeV}}{20} = 2 \times 50\, \Delta\phi .$$

Thus

$$\Delta\phi \sim \frac{0.14}{2000} = 0.07 \text{ mrads} .$$

This is almost exactly the same as necessary for reasonable mass resolution!

Angular Resolution of the Apparatus

Necessary angular resolution was shown to be of the order of 0.07 mrads. If wire planes are used to determine the position of the particle, these are capable of something like 0.25 mm resolution. The necessary distance D, between planes is

$$D = \frac{0.25\,\mathrm{mm}}{0.00007} \sqrt{2} = 5\ \mathrm{m}.$$

The factor of $\sqrt{2}$ comes from the two independent positions necessary to determine the angle.

Lateral Size of the Magnet

In order to let the K^o decay over one-mean free path, a drift space of 5.4 m is necessary at 100 GeV. The target must be placed, then, ~ 5 m from the first wire plane which is in turn 5 m from the magnet.

One of the pions can depart from the direction of the K^o by about 8 mrads. The K^o in turn can go out to angles of

$$\frac{4 \times 0.14}{100} = 5.6\ \mathrm{mrads},$$

from the forward direction, if momentum transfers of 4 pion masses are to be measured. All the pions are contained in a cone of 13.6 mrads which subtends a distance of 13.6 cm at a distance of 10 m. The magnet must, therefore, have an aperture of 27 cm or about 1 foot square.

The Magnet

It is probably unreasonable to construct a magnet as small as seems necessary. A fairly reasonable compromise useful for ρ-mesons, etc., would be a magnet 24 in. $\times$ 24 in. $\times$ 48 in. long with a field of 15 kG. To take care of the lateral bending of low-momentum particles it should probably be somewhat wider than tall--say 30 in. wide by 48 in. long with a gap of 24 in.

Lower Energies

Lower-energy reactions are taken care of by moving the target closer to the magnet--everything scales accordingly.

Triggering Requirements

In order to actually perform this experiment a suitable trigger must be devised. A pion incident followed by nothing, i.e. no charged particle, out followed by two charged particles through the magnet would probably be sufficient, but might be bedevilled by π^- charge exchange with the γ-ray converting. The addition of a Λ requirement would almost certainly reduce the trigger rate to a tractable value.

Difficulties

At 8 GeV $\Lambda^o K^o + n(\pi^o)$ channels are twice as large as the $\Lambda^o K^o$ channel. This does not seem an insurmountable background to handle with π^o anti-counters around the target. Such an anti would also handle the $\Sigma^o K^o$ reaction if it were very efficient.

The Λ's produced in this experiment are very slow, varying from 0.2 GeV at forward angles to 0.6 GeV at 50° to the forward direction. Many of these decay before leaving the target and any anticoincidence scheme must be very thoroughly understood so that the anticoincidence does not invalidate the angular distribution. This may be such a serious question as to prevent the experiment from being done in this way.

Necessary Equipment and Facilities

The experiment is conceived of as being done with wire spark chambers. The array for detecting the decaying K^0's is very straightforward, that for the Λ definitely quite complicated. If logical reduction of the data is done on-line, a fairly sophisticated computer will be necessary. One possibility is that high enough rates can be handled so that triggering can be done on the K^0 only and events selected on the basis of computation. At any rate, the experiment is envisoned as using an on-line computer.

The magnet, even though it is not a very large piece of equipment, is special enough to warrant mention.

Aside from these, the principal interest in this discussion seems to me to be that a reasonable experiment can be performed with very little in the way of special NAL-supplied equipment.

The experimental layout is shown in Fig. 1. A space about 60 ft long by 10 ft wide is needed for the experiment plus space for one large or two small trailers and shields and beam stop.

Detecting Extra Neutral Pions by Measuring the K^o Energy Precisely

Considerable discussion arose concerning the modest scheme proposed in the previous sections. The chief questions concerned the practicability of measuring the K^o energy itself to sufficient accuracy to detect a missing π^o. The difference between the energy of the incoming π^- and the outgoing K^o must be known to one pion mass, 0.140 GeV or 0.14 percent of the total energy of 100 GeV. The incoming pion beam energy can only be defined to about 0.1 percent so that the outgoing K^o energy must be known also to 0.1 percent. Subject to a few simple approximations it can be shown that the relative error in the energy is

$$\frac{\Delta E}{E} = 2 \times \frac{100}{3} \frac{p}{BL} \frac{\delta}{S},$$

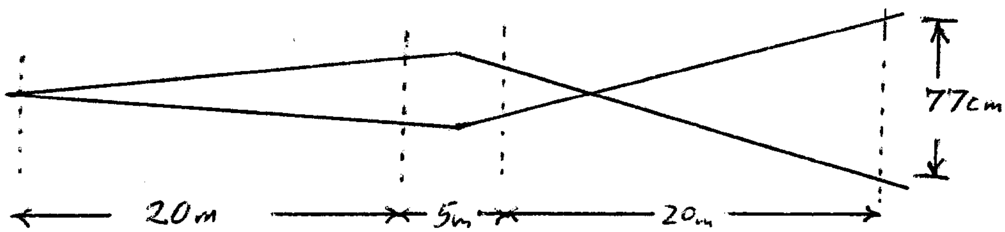
where p is the momentum of the K^o, BL is the field times the path length through the bending magnet, and δ is the accuracy to which a spark position can be measured in chambers a distance S apart before the magnet and after the magnet. If p = 100 GeV and δ = 0.25 mm and $\Delta E/E = 0.001$

$$BLS = 1666 \text{ kGm}^2.$$

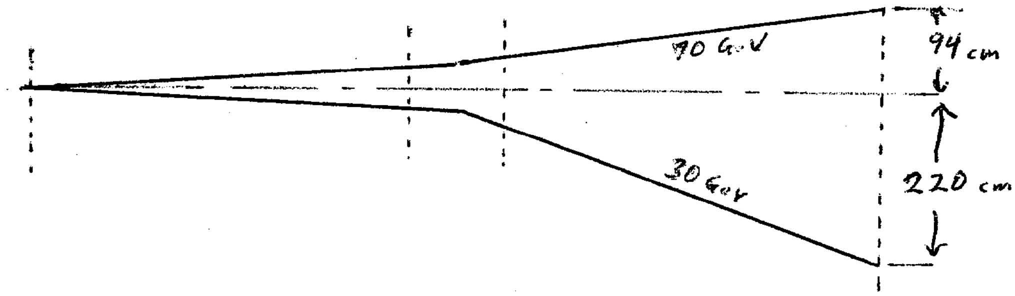
There is the obvious choice of a powerful magnet or long measuring distance. A reasonable choice, though not necessarily optimum, is

S = 20 m, L = 5 m, B = 17 kG. As before, the widest-angle pion goes at 13.6 mrads. Using a decay space of 5 m, distance between chambers of 20 m, and magnet length of 5 m, gives a necessary height of twice 41 cm or 81 cm. The magnets' width must be somewhat greater, perhaps 125 cm. This magnet is much larger than that required by the previous scheme, but it is not impossibly large.

A decay in which each pion has 50 GeV has the following topology:



A decay at 60° in the center-of-mass gives approximately 70 GeV to one pion and 30 GeV to the other. The "worst" topology looks like:



The last chamber must be 4.5 m across and, on each firing, the position of a spark must be known to 0.25 mm, an accuracy of 1 part in 18,000. The recording apparatus must have 15-bit accuracy--which exceeds most present-day systems, but seems possible.

Absolute accuracy could probably not be maintained at this level, but by periodically turning off the magnet a straight-through track can be recorded to determine lineup, and then a beam track recorded with the magnet on to determine the relative energy of beam and decay particles. It should be emphasized that this is not the usual case of a single-arm spectrometer that is usually self-calibrating against an elastically-scattered particle. Two sources of error have been ignored: magnet calibration and multiple scattering. The latter becomes a major error source if 0.005 radiation lengths of material are crossed before the magnet. This is about 40 m of He, so that multiple scattering is just barely smaller than other effects; it will significantly increase the error. The integral of BL can probably be calibrated to 0.1 percent--but with difficulty.

In summary, the direct way looks very hard! It does not look like a setup for early experiments with the machine!

REFERENCES

[1]A. Roberts, Preliminary Considerations on the Characteristics of a Large Spark-Chamber Detecting System for Complex High-Energy Events, 200 BeV Accelerator: Studies on Experimental Use, Vol. 3, 1966.

[2]N. H. Lipman and T. G. Walker, Large Volume Magnet Spark Chamber for Operation Up to 300 GeV, Utilization Studies for a 300-GeV Proton Synchrotron, Vol. I, 1967.

[3]Ehrlich, Selove and Yuta, Phys. Rev. 152, 1194 (1966).

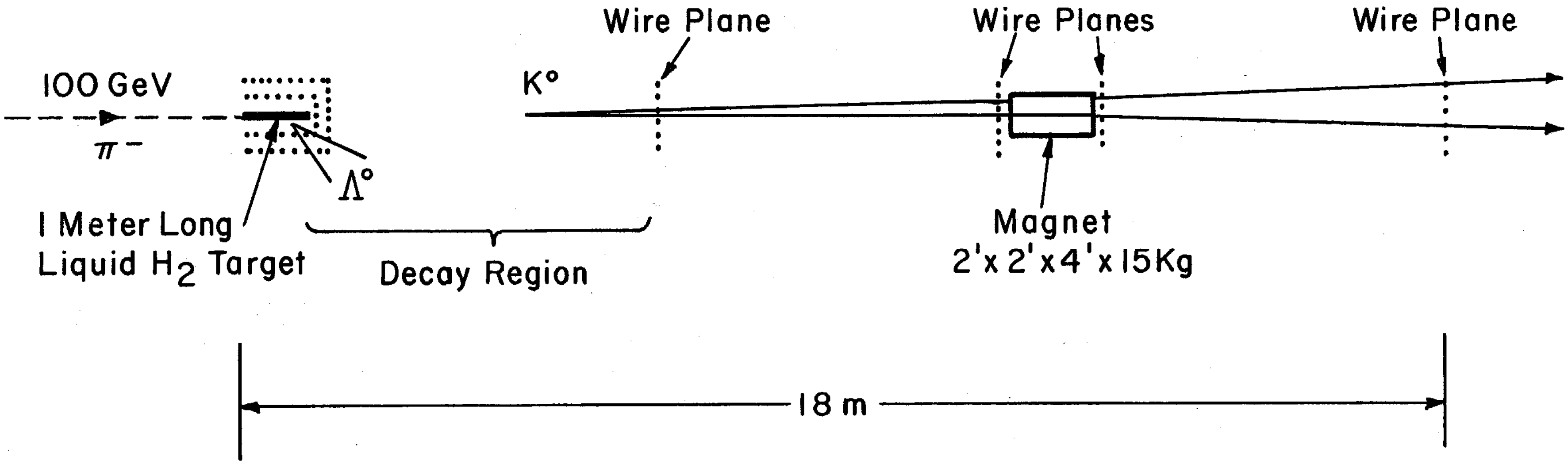


Fig. 1. Experiment on associated production at 100 GeV/c. Slow lambdas, fast K^o's are assumed. Extra neutral pions must be eliminated by anticoincidence counters or chambers (not shown).

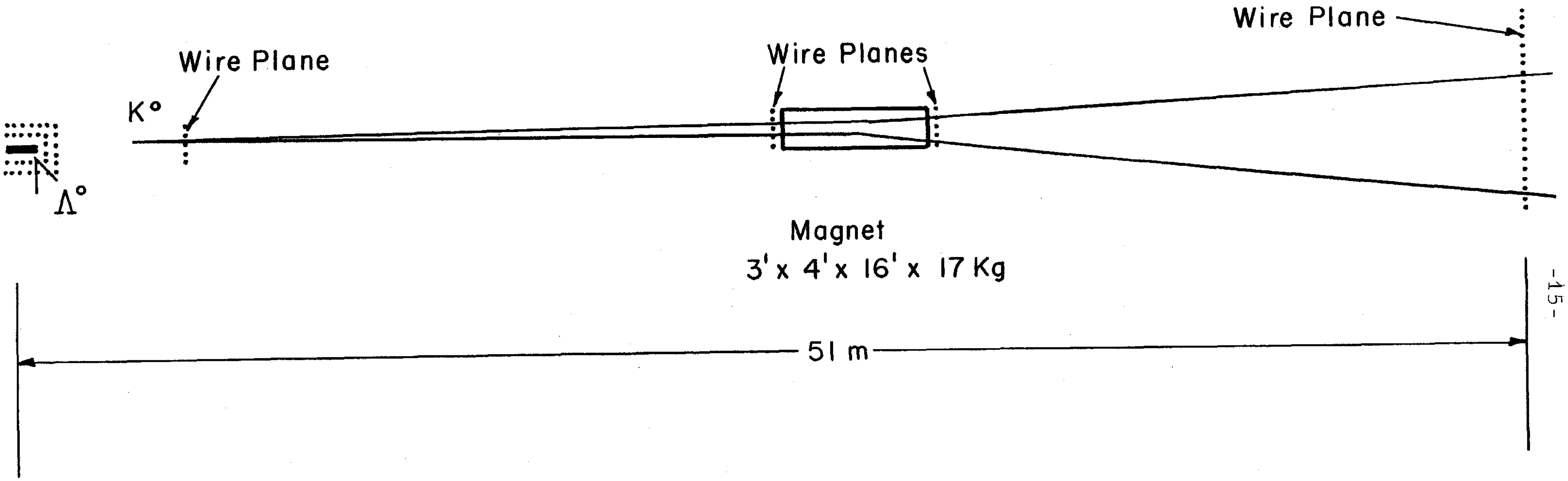


Fig. 2. An associated production experiment in which the energy of the K^o is determined with sufficient accuracy to detect a missing neutral pion.